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Internal W-emission and W-exchange Contributions to $B \to D^{(*)}$ Decays

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Abstract

We evaluate external W-emission, internal W-emission and W-exchange contributions to nonleptonic $B \to D^{(*)}$ decays based on the perturbative QCD formalism including Sudakov effects, whose ratio is found to be $1 : +0.2 : 0.03i$ at the amplitude level. We observe that the internal W-emission contribution is additive to the external W-emission contribution, and that the W-exchange contribution is negligible and mainly imaginary, its real part being at least one order of magnitude smaller than the imaginary part. Our predictions are consistent with the CLEO data and with those obtained by the Bauer-Stech-Wirbel method.

1. Introduction

The conventional approach to nonleptonic B meson decays is the Bauer-Stech-Wirbel (BSW) method [1], in which two parameters a_1 and a_2 are associated with the external and internal W-emission amplitudes, respectively, and are determined by experimental data. a_1 and a_2 were originally the linear combination of the Wilson coefficients c_1 and c_2 in the effective Hamiltonian

$$H_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* [c_1(\mu)(\bar{d}u)(\bar{c}b) + c_2(\mu)(\bar{c}u)(\bar{d}b)] \;, \tag{1}$$

written as $a_1 = c_1 + c_2/N_c$ and $a_2 = c_2 + c_1/N_c$, N_c being the number of colors. $H_{\rm eff}$ was derived from the Hamiltonian

$$H = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* (\bar{d}u)(\bar{c}b) \;, \tag{2}$$

with hard gluon corrections taken into account by renormalization group (RG) methods. Here $(\bar{q}_i q_j) = \bar{q}_i \gamma_\mu (1-\gamma_5) q_j$ represents the $V-A$ current, G_F is the Fermi coupling constant, and V_{cb} and V_{ud} are the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements.

The hadronic form factors involved in nonleptonic B meson decays are usually assumed to take, say, a monopole or dipole ansatz [2], and thus the

extraction of a_1 and a_2 is model dependent. It has been found that the ratio a_2/a_1 from an individual fit to the CLEO data of $B \to D^{(*)}\pi(\rho)$ [3] varies significantly [2]. It has even been shown that an allowed domain (a_1, a_2) exists for the three classes of decays $\bar{B}^0 \to D^{(*)+}$, $\bar{B}^0 \to D^{(*)0}$ and $B^- \to D^{(*)0}$, only when the experimental errors are expanded to a large extent [4]. On the other hand, a negative a_2/a_1 and a positive a_2/a_1 were concluded from the data of charm and bottom decays [1, 5], respectively, and this subject remains controversial.

In the BSW model nonspectator contributions from W-exchange (or annihilation) diagrams are not included. In previous studies such contributions were assumed to be negligible, though a convincing justification is not yet available. Hence, it has been a challenging task to evaluate W-exchange contributions to nonleptonic B meson decays reliably.

Recently, we have shown that the perturbative QCD (PQCD) formalism, which contains the resummation of large radiative corrections, is applicable to $B \to D$ decays in the fast recoil region of the D meson [6], and have employed this formalism to investigate the spectator contributions from external W-emission diagrams to $B \to D^{(*)}$ decays [7]. In our approach all the relevant form factors can be evaluated explicitly without resort to models.

For neutral B meson decays such as $\bar{B}^0 \to D^{(*)+}\pi^-(\rho^-)$, there are additional contributions from W-exchange diagrams. For charged B meson decays such as $B^- \to D^{(*)0}\pi^-(\rho^-)$, both external and internal W-emission diagrams contribute.

In this letter we shall extend the PQCD formalism to the study of internal W-emission and W-exchange diagrams, whose contributions are computed reliably. Our work helps clarify the ambiguity in the extraction of a_1 and a_2 in the BSW model. A PQCD analysis of these contributions based on the Brodsky-Lepage theory [8] has been performed in [9]. However, this approach was criticized in [10]: The parametrization of the fractional momentum carried by the light valence quark in a B meson does not satisfy the on-shell requirement from the parton model, such that extra imaginary contributions are introduced. Moreover, the predictions depend strongly on the model of meson wave functions [9, 10].

2. Resummation

We review the investigation of the external W-emission diagrams as shown in Fig. 1(a), which involve six form factors ξ_i, $i = +, -, V, A_1, A_2$ and A_3, de-

fined by the hadronic matrix elements of vector and axial vector currents [7]. Assume that the momentum P_1 (P_2), the mass M_B ($M_{D^{(*)}}$) and the velocity v_1 (v_2) of the B ($D^{(*)}$) meson are related by $P_1 = M_B v_1$ ($P_2 = M_{D^{(*)}} v_2$). The velocity transfer $\eta = v_1 \cdot v_2$ is expressed in terms of the momentum transfer $q^2 = (P_1 - P_2)^2$ as

$$\eta = \frac{M_B^2 + M_{D^{(*)}}^2 - q^2}{2 M_B M_{D^{(*)}}} \; . \tag{3}$$

In the infinite mass limit of M_B and $M_{D^{(*)}}$, the six form factors have the relations

$$\xi_+ = \xi_V = \xi_{A_1} = \xi_{A_3} = \xi, \quad \xi_- = \xi_{A_2} = 0. \tag{4}$$

ξ is the so-called Isgur-Wise (IW) function [11], which is normalized to unity at zero recoil $\eta \to 1$ by heavy quark symmetry (HQS).

In the rest frame of the B meson P_1 is written as $P_1 = (M_B/\sqrt{2})(1, 1, \mathbf{0}_T)$, and P_2 has the nonvanishing components [6]

$$P_2^+ = \frac{\eta + \sqrt{\eta^2 - 1}}{\sqrt{2}} M_{D^{(*)}} \; , \quad P_2^- = \frac{\eta - \sqrt{\eta^2 - 1}}{\sqrt{2}} M_{D^{(*)}} \; . \tag{5}$$

Let k_1 (k_2) be the momentum of the light valence quark in the B ($D^{(*)}$) meson, satisfying $k_1^2 \approx 0$ ($k_2^2 \approx 0$). k_1 may have a large minus component k_1^-, defining the momentum fraction $x_1 = k_1^-/P_1^-$, and small transverse components $\mathbf{k}_{1T}$. k_2 may have a large plus component k_2^+, defining $x_2 = k_2^+/P_2^+$,

and small $\mathbf{k}_{2T}$. As $\eta \to 1$ with $P_2^+ = P_2^- = M_{D^{(*)}}/\sqrt{2}$, the two meson wave functions strongly overlap, and the form factors ξ_i are dominated by soft contributions. In the large η limit with $P_2^+ \gg M_{D^{(*)}}/\sqrt{2} \gg P_2^-$, the $D^{(*)}$ meson behaves like a light meson [6]. Then $B \to D^{(*)}$ transitions occur through hard gluon exchanges, to which PQCD is applicable.

Double logarithms from radiative corrections to the B ($D^{(*)}$) meson wave function ϕ_B ($\phi_{D^{(*)}}$) have been organized into an exponent s using the resummation technique [6]. We quote the results as

$$\begin{aligned}
\phi_B(x_1, P_1, b_1, \mu) &= \phi_B(x_1) \exp\left[-s(x_1 P_1^-, b_1) - 2\int_{1/b_1}^{\mu} \frac{d\bar{\mu}}{\bar{\mu}} \gamma(\alpha_s(\bar{\mu}))\right] , \\
\phi_{D^{(*)}}(x_2, P_2, b_2, \mu) &= \phi_{D^{(*)}}(x_2) \exp\Big[-s(x_2 P_2^+, b_2) - s((1-x_2)P_2^+, b_2) \\
&\quad -2\int_{1/b_2}^{\mu} \frac{d\bar{\mu}}{\bar{\mu}} \gamma(\alpha_s(\bar{\mu}))\Big] , \qquad (6)
\end{aligned}$$

where b_1 (b_2) is the Fourier conjugate variable of k_{1T} (k_{2T}), and can be regarded as the spatial extent of the B ($D^{(*)}$) meson. μ is a renormalization and factorization scale. $\gamma = -\alpha_s/\pi$ is the quark anomalous dimension. The expression of s is very complicated and referred to [6, 7]. The initial conditions $\phi_i(x)$, $i = B$, D and D^*, are of nonperturbative origin, satisfying the normalization $\int_0^1 \phi_i(x)dx = f_i/(2\sqrt{6})$, f_i being the corresponding meson decay constants.

The evolution of the hard scattering amplitude H is expressed as [6]

$$H(k_1^-, k_2^+, b_1, b_2, \mu) = H(k_1^-, k_2^+, b_1, b_2, t) \exp\left[-4\int_\mu^t \frac{d\bar{\mu}}{\bar{\mu}}\gamma(\alpha_s(\bar{\mu}))\right] , \qquad (7)$$

where the variable t denotes the largest mass scale of H. Combining Eqs. (6) and (7), we obtain the factorization formulas of ξ_i.

For nonleptonic decays, there exist additional important corrections from final-state interactions with soft gluons attaching the outgoing hadrons. It has been argued that these corrections produce only single logarithms [6], and are thus not considered here.

3. Nonleptonic $B \to D^{(*)}$ decays

In the analysis of nonleptonic B meson decays we employ the Hamiltonian in Eq. (2), instead of the effective Hamiltonian in (1). We shall address how to formulate our PQCD theory based on the effective Hamiltonian in the end of this letter. The amplitudes for charged and neutral B meson decays are then written as

$$\frac{G_F}{\sqrt{2}}V_{cb}V_{ud}^*\left[\langle\pi^-|(\bar{d}u)|0\rangle\langle D^{(*)0}|(\bar{c}b)|B^-\rangle - \langle\pi^-|\langle D^{(*)0}|(\bar{d}u)(\bar{c}b)|B^-\rangle\right], \qquad (8)$$

$$\frac{G_F}{\sqrt{2}}V_{cb}V_{ud}^*\left[\langle\pi^-|(\bar{d}u)|0\rangle\langle D^{(*)+}|(\bar{c}b)|\bar{B}^0\rangle - \langle\pi^-|\langle D^{(*)+}|(\bar{d}u)(\bar{c}b)|\bar{B}^0\rangle\right]. \qquad (9)$$

The first terms in Eqs. (8) and (9) correspond to the external W-emission contributions, and the second terms to the internal W-emission and W-exchange contributions, respectively. Note the minus signs in front of the second terms, which are associated with the interchange of two u (d) quarks in the internal W-emission (W-exchange) diagrams as shown in Fig. 1(b) (Fig. 1(c)) compared to Fig. 1(a). There are another two nonfactorizable internal W-emission diagrams, in which one end of the gluon line attaches the quark of the $D^{(*)}$ meson. However, it can be shown that the contributions from these two diagrams cancel partially, and are thus less important. For the same reason, we neglect the contributions from another two nonfactorizable W-exchange diagrams, in which the gluon attaches the quark of the B meson.

It has been argued that the internal W-emission and W-exchange amplitudes can be evaluated in the PQCD approach [7]. The decay rates of $B \to D^{(*)}$ transitions have the expression

$$\Gamma_i = \frac{1}{128\pi} G_F^2 |V_{cb}|^2 |V_{ud}|^2 m_B^3 \frac{(1-r^2)^3}{r} |\mathcal{M}_i|^2 \ , \tag{10}$$

with $r = M_{D^{(*)}}/M_B$ and i = 1, 2, 3 and 4 denoting $B^- \to D^0\pi^-$, $\bar{B}^0 \to D^+\pi^-$, $B^- \to D^{*0}\pi^-$ and $\bar{B}^0 \to D^{*+}\pi^-$, respectively. The amplitudes $\mathcal{M}_i$

are written as

$$
\begin{aligned}
\mathcal{M}_1 &= f_\pi[(1+r)\xi_+ - (1-r)\xi_-] + \frac{f_D}{N_c}\xi_{\rm int} \;, && (11)\\
\mathcal{M}_2 &= f_\pi[(1+r)\xi_+ - (1-r)\xi_-] + \frac{\pi^2}{4N_c} f_B \xi_{\rm exc} \;, && (12)\\
\mathcal{M}_3 &= \frac{1+r}{2r} f_\pi[(1+r)\xi_{A_1} - (1-r)(r\xi_{A_2}+\xi_{A_3})] + \frac{f_{D^*}}{N_c}\xi^*_{\rm int} \;, && (13)\\
\mathcal{M}_4 &= \frac{1+r}{2r} f_\pi[(1+r)\xi_{A_1} - (1-r)(r\xi_{A_2}+\xi_{A_3})] + \frac{\pi^2}{4N_c} f_B \xi^*_{\rm exc} \;. && (14)
\end{aligned}
$$

The color-suppressing factors $1/N_c$ have been shown explicitly.

The factorization formulas for ξ_i, $i = +$, V, A_1 and A_3 ($\xi_{A_3} = \xi_V$) and for ξ_j, $j = -$ and A_2 are given by [7]

$$
\begin{aligned}
\xi_i &= 16\pi\mathcal{C}_F\sqrt{r}M_B^2\int_0^1 dx_1dx_2\int_0^\infty b_1db_1b_2db_2\,\phi_B(x_1)\phi_{D^{(*)}}(x_2)\alpha_s(t)\\
&\quad\times[(1+\zeta_i x_2 r)h(x_1,x_2,b_1,b_2,m) + (r+\zeta'_i x_1)h(x_2,x_1,b_2,b_1,m)]\\
&\quad\times\exp[-S(x_1,x_2,b_1,b_2)] \;, && (15)\\
\xi_j &= 16\pi\mathcal{C}_F\sqrt{r}M_B^2\int_0^1 dx_1dx_2\int_0^\infty b_1db_1b_2db_2\,\phi_B(x_1)\phi_{D^{(*)}}(x_2)\alpha_s(t)\\
&\quad\times[\zeta_j x_2 r h(x_1,x_2,b_1,b_2,m) + \zeta'_j x_1 h(x_2,x_1,b_2,b_1,m)]\\
&\quad\times\exp[-S(x_1,x_2,b_1,b_2)] \;, && (16)
\end{aligned}
$$

with the constants

$$
\zeta_+ = \zeta'_+ = \frac{1}{2}\left[\eta - \frac{3}{2} + \sqrt{\frac{\eta-1}{\eta+1}}\left(\eta-\frac{1}{2}\right)\right] \;,
$$

$$
\begin{aligned}
\zeta_V &= -\frac{1}{2} - \frac{\eta - 2}{2\sqrt{\eta^2-1}} \;, \quad \zeta'_V = \frac{1}{2\sqrt{\eta^2-1}} \;, \\
\zeta_{A_1} &= -\frac{2-\eta-\sqrt{\eta^2-1}}{\eta+1} \;, \quad \zeta'_{A_1} = \frac{1}{2(\eta+1)} \;, \\
\zeta_- &= -\zeta'_- = -\frac{1}{2}\left[\eta - \frac{1}{2} + \sqrt{\frac{\eta+1}{\eta-1}}\left(\eta - \frac{3}{2}\right)\right] \;, \\
\zeta_{A_2} &= 0 \;, \quad \zeta'_{A_2} = -1 - \frac{\eta}{\sqrt{\eta^2-1}} \;.
\end{aligned} \tag{17}
$$

$\mathcal{C}_F = 4/3$ is the color factor. These form factors are evaluated at the maximal recoil $\eta = \eta_{\rm max} = (1+r^2)/(2r)$ in Eqs. (11) to (14).

The form factors $\xi^{(*)}_{\rm int}$ and $\xi^{(*)}_{\rm exc}$ are given by

$$
\begin{aligned}
\xi^{(*)}_{\rm int} &= 16\pi\mathcal{C}_F\sqrt{r}M_B^2\int_0^1 dx_1dx_3\int_0^\infty b_1db_1b_3db_3\phi_B(x_1)\phi_\pi(x_3)\alpha_s(t_{\rm int}) \\
&\quad\times\Big[(1+x_3(1-r^2))h_{\rm int}(x_1,x_3,b_1,b_3,m_{\rm int}) \\
&\qquad+\zeta^{(*)}_{\rm int}x_1r^2h_{\rm int}(x_3,x_1,b_3,b_1,m_{\rm int})\Big]\exp[-S_{\rm int}(x_1,x_3,b_1,b_3)] \;, \qquad (18) \\
\xi^{(*)}_{\rm exc} &= 16\pi\mathcal{C}_F\sqrt{r}M_B^2\int_0^1 dx_2dx_3\int_0^\infty b_2db_2b_3db_3\phi_{D^{(*)}}(x_2)\phi_\pi(x_3)\alpha_s(t_{\rm exc}) \\
&\quad\times\Big[(x_3(1-r^2)-\zeta^{(*)}_{\rm exc}r^2)h_{\rm exc}(x_2,x_3,b_2,b_3,m_{\rm exc}) \\
&\qquad-x_2h_{\rm exc}(x_3,x_2,b_3,b_2,m_{\rm exc})]\exp[-S_{\rm exc}(x_2,x_3,b_2,b_3)] \;, \qquad (19)
\end{aligned}
$$

with the constants $\zeta_{\rm int} = -\zeta^*_{\rm int} = 1$ and $\zeta_{\rm exc} = -\zeta^*_{\rm exc} = 1$. In the derivation of $\xi^{(*)}_{\rm int}$ we have assumed that k_1 has a large plus component $k_1^+ = x_1P_1^+$. Here x_3 is the momentum fraction associated with the pion, and b_3 can be regarded as the spatial extent of the pion. $\xi^{(*)}_{\rm int}$ and $\xi^{(*)}_{\rm exc}$ are in fact the $B \to \pi$

and $D \to \pi$ transition form factors, respectively, evaluated at $\eta = \eta_{\rm max}$.

The Sudakov exponents S's, which group the exponents in Eqs. (6) and (7), are given by

$$
\begin{aligned}
S &= s(x_1P_1^-, b_1) + s(x_2P_2^+, b_2) + s((1-x_2)P_2^+, b_2) \\
&\quad -\frac{1}{\beta_1}\left[\ln\frac{\ln(t/\Lambda)}{-\ln(b_1\Lambda)} + \ln\frac{\ln(t/\Lambda)}{-\ln(b_2\Lambda)}\right] \qquad (20)\\
S_{\rm int} &= s(x_1P_1^+, b_1) + s(x_3P_3^-, b_3) + s((1-x_3)P_3^-, b_3) \\
&\quad -\frac{1}{\beta_1}\left[\ln\frac{\ln(t_{\rm int}/\Lambda)}{-\ln(b_1\Lambda)} + \ln\frac{\ln(t_{\rm int}/\Lambda)}{-\ln(b_3\Lambda)}\right]\;, \qquad (21)\\
S_{\rm exc} &= s(x_2P_2^+, b_2) + s((1-x_2)P_2^+, b_2) + s(x_3P_3^-, b_3) \\
&\quad +s((1-x_3)P_3^-, b_3) -\frac{1}{\beta_1}\left[\ln\frac{\ln(t_{\rm exc}/\Lambda)}{-\ln(b_2\Lambda)} + \ln\frac{\ln(t_{\rm exc}/\Lambda)}{-\ln(b_3\Lambda)}\right]\;, \qquad (22)
\end{aligned}
$$

with $\beta_1 = (33 - 2n_f)/12$ and $n_f = 4$ the number of flavors. The QCD scale $\Lambda \equiv \Lambda_{\rm QCD}$ will be set to 0.2 GeV below. The factors e^{-S} fall off quickly in the large b, or long-distance, region, giving so-called Sudakov suppression.

In Eqs. (15), (16), (18) and (19) the functions h's, obtained from the Fourier transform of the lowest-order H, are given by

$$
\begin{aligned}
h(x_1, x_2, b_1, b_2, m) &= K_0\left(\sqrt{x_1x_2m}b_1\right) \\
&\quad \times\left[\theta(b_1-b_2)K_0\left(\sqrt{x_2m}b_1\right)I_0\left(\sqrt{x_2m}b_2\right)\right. \\
&\quad \left.+\theta(b_2-b_1)K_0\left(\sqrt{x_2m}b_2\right)I_0\left(\sqrt{x_2m}b_1\right)\right]\;, \qquad (23)
\end{aligned}
$$

$$h_{\rm int}(x_1,x_3,b_1,b_3,m_{\rm int}) = h(x_1,x_3,b_1,b_3,m_{\rm int}) \;, \tag{24}$$

$$\begin{aligned} h_{\rm exc}(x_2,x_3,b_2,b_3,m_{\rm exc}) &= H_0^{(1)}\left(\sqrt{x_2x_3m_{\rm exc}}b_2\right) \\ &\times\Big[\theta(b_2-b_3)H_0^{(1)}\left(\sqrt{x_3m_{\rm exc}}b_2\right)J_0\left(\sqrt{x_3m_{\rm exc}}b_3\right) \\ &+\theta(b_3-b_2)H_0^{(1)}\left(\sqrt{x_3m_{\rm exc}}b_3\right)J_0\left(\sqrt{x_3m_{\rm exc}}b_2\right)\Big] \;, \end{aligned} \tag{25}$$

with $m = (\eta+\sqrt{\eta^2-1})M_BM_{D^{(*)}}$ and $m_{\rm int} = m_{\rm exc} = M_B^2 - M_{D^{(*)}}^2$. It is obvious that the W-exchange contribution is complex due to the exchange of a time-like hard gluon. In the above expressions the large scales t's take the values

$$t = \max(\sqrt{x_1x_2m}, 1/b_1, 1/b_2) \tag{26}$$

$$t_{\rm int} = \max(\sqrt{x_1x_3m_{\rm int}}, 1/b_1, 1/b_3) \tag{27}$$

$$t_{\rm exc} = \max(\sqrt{x_2x_3m_{\rm exc}}, 1/b_2, 1/b_3) \;. \tag{28}$$

We choose $f_B = 200$ MeV, $f_D = f_{D^*} = 220$ MeV [3], $|V_{cb}| = 0.043$ for the CKM matrix element [6, 7], and the Chernyak-Zhitnitsky model [12]

$$\phi_\pi(x) = \frac{5\sqrt{6}}{2}f_\pi x(1-x)(1-2x)^2 \tag{29}$$

for the pion wave function, $f_\pi = 132$ MeV being the pion decay constant. For the B meson, we employ the wave function from the relativistic constituent

quark model [13],

$$\phi_B(x,\mathbf{k}_T) = N_B\left[C_B + \frac{M_B^2}{1-x} + \frac{k_T^2}{x(1-x)}\right]^{-2} . \tag{30}$$

The normalization constant N_B and the shape parameter C_B are determined by the conditions

$$\begin{aligned}
\int_0^1 dx \int \frac{d^2\mathbf{k}_T}{16\pi^3}\phi_B(x,\mathbf{k}_T) &= \frac{f_B}{2\sqrt{6}} , \\
\int_0^1 dx \int \frac{d^2\mathbf{k}_T}{16\pi^3}[\phi_B(x,\mathbf{k}_T)]^2 &= \frac{1}{2} .
\end{aligned} \tag{31}$$

The B meson wave function is then given by

$$\phi_B(x) = \int \frac{d^2\mathbf{k}_T}{16\pi^3}\phi_B(x,\mathbf{k}_T) = \frac{N_B}{16\pi^2}\frac{x(1-x)^2}{M_B^2 + C_B(1-x)} , \tag{32}$$

with $N_B = 650.212$ and $C_B = -27.1051$. We assume that the $D^{(*)}$ meson wave function possesses the same functional form as Eq. (32),

$$\phi_{D^{(*)}}(x) = \frac{N_{D^{(*)}}}{16\pi^2}\frac{x(1-x)^2}{M_{D^{(*)}}^2 + C_{D^{(*)}}(1-x)} , \tag{33}$$

but the shape parameter $C_D^{(*)}$ is fixed by the $B \to D^{(*)}$ data [3].

4. Discussion

We adopt $G_F = 1.16639 \times 10^{-5}$ GeV^{-2}, $|V_{ud}| = 0.974$, $M_B = 5.28$ GeV, $M_D = 1.87$ GeV, $M_{D^*} = 2.01$ GeV [14], and $\tau_{B^0} = 1.53$ ($\tau_{B^-} = 1.68$) ps for

the $\bar{B}^0$ (B^-) meson lifetime [15]. The experimental data of the branching ratios are $\mathcal{B}(B^- \to D^0\pi^-) = (5.5 \pm 1.1) \times 10^{-3}$, $\mathcal{B}(\bar{B}^0 \to D^+\pi^-) = (2.9 \pm 1.2) \times 10^{-3}$, $\mathcal{B}(B^- \to D^{*0}\pi^-) = (5.2 \pm 1.7) \times 10^{-3}$, and $\mathcal{B}(\bar{B}^0 \to D^{*+}\pi^-) = (2.6 \pm 0.8) \times 10^{-3}$ [3]. We determine the parameter $C_D^{(*)}$, *ie.*, the $D^{(*)}$ meson wave function, by fitting our predictions to $\mathcal{B}(\bar{B}^0 \to D^+\pi^-)$ and $\mathcal{B}(\bar{B}^0 \to D^{*+}\pi^-)$. They are found to be $C_D = -2.95$ GeV^2 and $C_{D^*} = -3.05$ GeV^2. The corresponding normalization constants are then $N_D = 128.77$ GeV^3 and $N_{D^*} = 173.48$ GeV^3.

With these parameters we derive the branching ratios $\mathcal{B}(B^- \to D^0\pi^-) = 4.41 \times 10^{-3}$, $\mathcal{B}(\bar{B}^0 \to D^+\pi^-) = 2.91 \times 10^{-3}$, $\mathcal{B}(B^- \to D^{*0}\pi^-) = 3.97 \times 10^{-3}$, and $\mathcal{B}(\bar{B}^0 \to D^{*+}\pi^-) = 2.59 \times 10^{-3}$, which are within the errors of the data. It is observed that the internal W-emission amplitude is additive to and about 20% of the external W-emission amplitude. Therefore, our analysis favors a positive a_2/a_1 for B meson decays. The internal W-emission contribution obtained in [9] is complex, but it is real in our formalism, and is twice larger in magnitude. Hence, our predictions for the charged B meson decays are in a better agreement with the data. We also find that the W-exchange amplitude is mainly imaginary, and its magnitude is only 3% of the external W-emission amplitude. The real part of the W-exchange contribution is at least one

order of magnitude smaller than the imaginary part. The corresponding results in [9] have the same order, but the real part is larger or only slightly smaller than the imaginary part in magnitude for different models of pion wave functions. The branching ratios of the charged B meson decays are then $(1.2)^2 \times (\tau_{B^-}/\tau_{B^0}) \approx 1.5$ times of those of the neutral B meson decays, fairly consistent with the data, and with the prediction $\mathcal{B}(B^- \to D^{(*)0}\pi^-)/\mathcal{B}(\bar{B}^0 \to D^{(*)+}\pi^-) \approx 1.7$ from the BSW method [3].

The branching ratios of the charged B meson decays fall below the central values of the data. The agreement can be improved by including the two nonfactorizable internal W-emission diagrams. Because of the complexity of their evaluation, we shall not study these two diagrams in this letter, but discuss them elsewhere.

It is worthwhile to exhibit the spectrum $d\Gamma/dq^2$ of the semileptonic decay $\bar{B}^0 \to D^{*+}\ell^-\bar{\nu}$. For this process only the external W-emission diagrams contribute, and the expression is written as [7]

$$\begin{aligned}
\frac{d\Gamma}{dq^2} &= \frac{1}{96\pi^3}G_F^2|V_{cb}|^2 M_B^3 r^2(\eta^2-1)^{1/2}(\eta+1)^2 \\
&\times\Bigg\{2(1-2\eta r+r^2)\left[\xi_{A_1}^2(\eta)+\frac{\eta-1}{\eta+1}\xi_V^2(\eta)\right] \\
&\quad+\left[(\eta-r)\xi_{A_1}(\eta)-(\eta-1)\left(r\xi_{A_2}(\eta)+\xi_{A_3}(\eta)\right)\right]^2\Bigg\}\,, \qquad (34)
\end{aligned}$$

where the form factors $\xi_i(\eta)$ have been defined in Eqs.(15) and (16). It is observed from Fig. 2 that our predictions match the data [15] at low q^2, but begin to deviate above $q^2 = 4$ GeV^2, the slow recoil region in which PQCD is not reliable.

Since the W-exchange contributions are negligible, and the $B \to \pi$ and $B \to \rho$ form factors are roughly equal [16], the branching ratios of the decays $B \to D^{(*)}\rho$ can be easily estimated by substituting the ρ meson decay constant $f_\rho = 220$ MeV for f_π in the corresponding formulas. Assuming a vanishing ρ meson mass, we have $\mathcal{B}(B^- \to D^0\rho^-) = 1.2\%$, $\mathcal{B}(\bar{B}^0 \to D^+\rho^-) = 8.1 \times 10^{-3}$, $\mathcal{B}(B^- \to D^{*0}\rho^-) = 1.1\%$, and $\mathcal{B}(\bar{B}^0 \to D^{*+}\rho^-) = 7.2 \times 10^{-3}$. Compared to the data $(1.35 \pm 0.30)\%$, $(8.1 \pm 3.6) \times 10^{-3}$, $(1.68 \pm 0.58)\%$, and $(7.4 \pm 2.7) \times 10^{-3}$ [3], respectively. Our predictions are satisfactory except for the mode $B^- \to D^{*0}\rho^-$. Similarly, the overall consistency can be improved by including the two nonfactorizable internal W-emission diagrams.

It is nontrivial to incorporate the effective Hamiltonian in Eq. (1) into factorization theorems. The Wislson coefficients c_1 and c_2, or a_1 and a_2 equivalently, which take into account the evolution from the W boson mass M_W down to the scale μ, arise from the irreducible radiative corrections to the decay vertex. In the conventional approach μ is set to a value of order

M_b, and thus a scale dependence is introduced. We argue that μ in a_1 and a_2 should be set to the large scale t of the hard scattering. The scale t is then integrated over, such that predictions remain RG invariant. Hence, a_1 and a_2 are not parameters as in the BSW model, but vary according to their evolutions in our approach. The controversy over the scale setting in the analysis of inclusive nonleptonic B meson decays [17] may be resolved by our RG invariant formalism.

An immediate observation from the above choice of μ is that the internal W-emission amplitudes are still positive in bottom decays. If main contributions came from the region with t close to the b quark mass M_b, the coefficients of the external and internal W-emission amplitudes, $a_1 = 1$ and $a_2 = 1/N_c$, are replaced by $a_1(M_b) = 1.03$ and $a_2(M_b) = 0.11$. The internal W-emission amplitudes, however, may become negative for charm decays because of $a_1(M_c) = 1.09$ and $a_2(M_c) = -0.09$, M_c being the c quark mass. Certainly, this issue on the sign of a_2/a_1 [1, 5] needs more quantitative study. The details will be published in a separate work.

Effects from nonfactorizable final-state interactions [2] can be included into our formalism easily, which will be discussed elsewhere. The discrepancy between model-dependent theoretical predictions and experimental data as-

sociated with the decays $B \to \psi K^{(*)}$ [18] can also be investigated.

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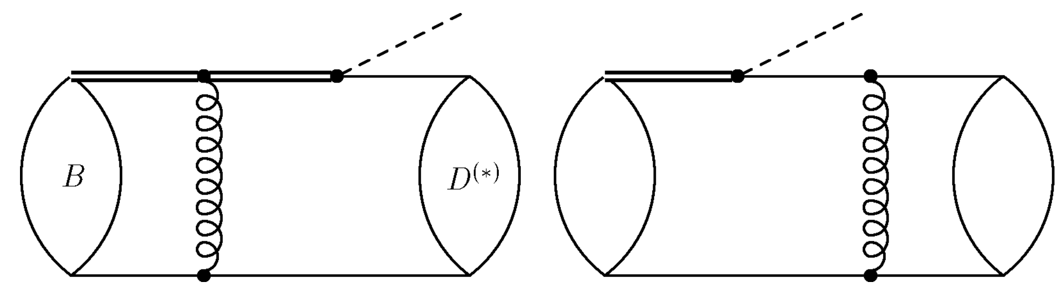
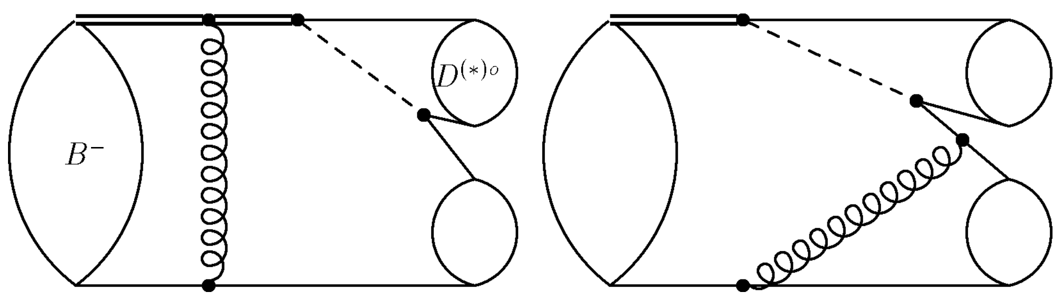
Figure Captions

Fig. 1. (a) External W-emission, (b) internal W-emission, and (c) W-exchange diagrams with the b quarks and the W bosons represented by double lines and dashed lines, respectively.

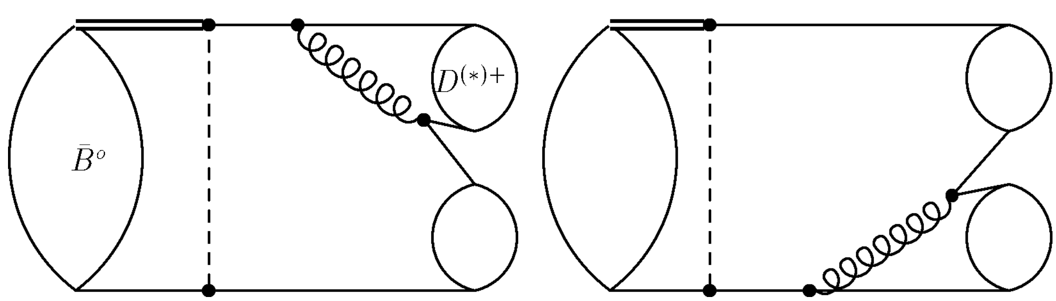
Fig. 2. The spectrum $d\Gamma/dq^2$ of the semileptonic decay $\bar{B}^0 \to D^{*+}\ell^-\bar{\nu}$.



(a)



(b)



(c)

Figure 1

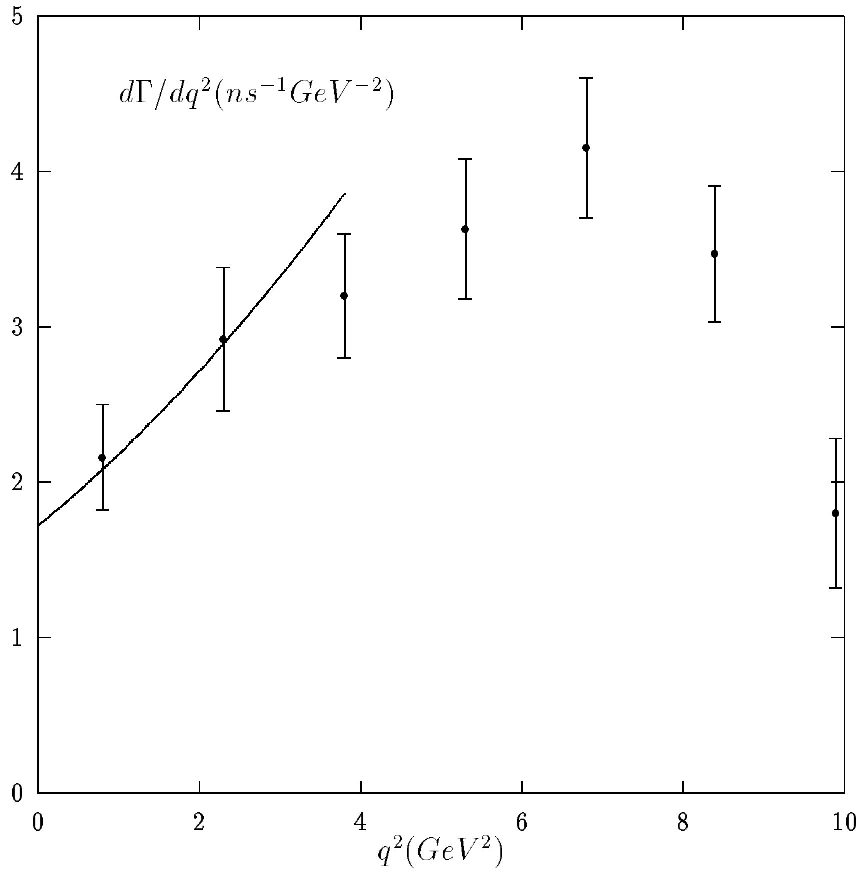


Figure 2